

# Tracing the Cosmic Web substructure with Lagrangian submanifold

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A new computational paradigm for the analysis of substructure of the Cosmic Web in cosmological cold dark matter simulations is proposed. We introduce a new data-field — the flip-flop field — which carries wealth of information about the history and dynamics of the structure formation in the universe. The flip-flop field is an ordered data set in Lagrangian space representing the number of sign reversals of an elementary volume of each collisionless fluid element represented by a computational particle in a  $N$ -body simulation. This field is computed using the Lagrangian submanifold, i.e., the three-dimensional dark matter sheet in the six-dimensional space formed by three Lagrangian and three Eulerian coordinates of the simulation particles. It is demonstrated that the very rich substructure of dark matter haloes and the void regions can be reliably and unambiguously recovered from the flip-flop field.

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Modern redshift surveys such as 2dF Galaxy Redshift Survey [1] and the Sloan Digital Sky Survey [2] and others reveal an intricate three dimensional structure in the spatial distribution of galaxies. Generic building blocks of the structure are: haloes, filaments, walls, and voids. A useful abstraction helping to visualize this structure has been provided by the adhesion approximation [3–5]. It is based on Burgers’ model of the equation of nonlinear diffusion [6]. In the limit of infinitesimal viscosity it approximates the structure by a geometrical construction made of two-dimensional curved faces, curvilinear edges at the surfaces crossings, and zero-dimensional vertices at the edge crossings. The vertices, edges and faces are analogous to haloes, filaments and walls respectively. Both the observations and the model exhibit large empty regions called by astronomers as voids.

Historically, haloes have attracted the most of attention in theoretical studies of the large-scale structure formation. From the observational point of view, haloes are most closely related to galaxies, galaxy groups and clusters of galaxies and they provide the bulk of information about the structures in the universe. However, direct modeling of galaxy formation based on fundamental laws of physics is precluded by enormous complexity of the physical processes involved, such as the highly nonlinear gravitational evolution of collisionless dark matter (DM) together with the hydrodynamical and thermal processes in baryons including star formation and the stellar wind feedback, shocks and supernovae explosions, gas accretion onto black holes in active galactic nuclei and the feedback via relativistic jets, and others. Hence various semi-empirical models of galaxy formation have been suggested, see e.g., [7] and references therein. In particular, it has been argued that galaxies are formed in the host DM haloes of corresponding masses. The DM haloes themselves are formed in a chain of mergers of smaller DM haloes which may start from tiny haloes

of a planet mass [8]. When two or more haloes merge their remnants may survive for a long time as subhaloes and/or streams within the resultant halo. Therefore, DM haloes are likely to have a hierarchical structure resembling a Russian doll or ‘matryoshka’, where each subhalo includes a number of even smaller subhaloes down to the smallest haloes allowed by the initial power spectrum [8, 9].

In early cosmological  $N$ -body simulations the haloes were loosely defined as compact concentrations of the simulation particles in configuration space. A particularly popular simple technique used for this purpose and called ‘friends of friends’ (FOF) algorithm was adopted from percolation analysis [10, 11]. According to this method one firstly finds all ‘friends’ of each particle by linking every particle in the simulation with all neighbors separated by less than a chosen distance – the linking length. Then applying the criterion: a friend of my friend is my friend, one can identify all groups of particles consisting of friends. Choosing a particular value of the linking length (often  $\sim 20\%$  of the mean particle separation) one can select a particular set of groups and call them haloes. Other more sophisticated methods that identify both haloes and subhaloes, some of which used only the positions of particle other also the phase space information, have been suggested as well, see e.g., [12–14] and references therein. The methods using only the configuration space information suffer from projection effect that causes dynamically distinct structures in phase space to overlap in configuration space (for illustration see e.g., fig. 1). Using all dynamical information provided by phase space is complicated by the fact that it is not a metric space [15].

In this letter we propose a novel technique which allows one to identify haloes and subhaloes by analyzing the mapping  $\mathbf{x} = \mathbf{x}(\mathbf{q}, t)$  where  $\mathbf{x}$  and  $\mathbf{q}$  are the coordinates of the particles in Eulerian and Lagrangian spaces respec-

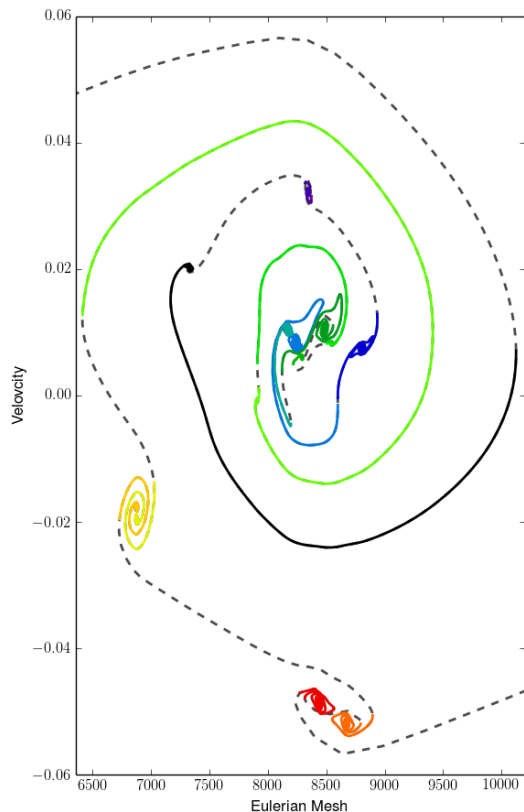


FIG. 1: The phase space of a one-dimensional halo simulated from random but smooth initial condition. The individual subhaloes are shown by different colors

tively. Topologically, this mapping, referred to as the Lagrangian submanifold, is a three-dimensional sheet in the six-dimensional  $(\mathbf{q}, \mathbf{x})$  space. The method is based on a concept of a DM sheet in phase space  $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$  successfully employed to improve accuracy of the estimates of the density, velocity and other parameters in standard cosmological  $N$ -body simulations [16, 17]. The major difference between this concept and the conventional one is in the different interpretation of the role of the particles in the simulations of the evolution of the continuous DM medium. Instead of the common interpretation of particles as carriers of mass, it was suggested to treat them as massless markers of the vertices in a tessellation of the three-dimensional DM sheet in six-dimensional phase space. The particles' mass is uniformly distributed inside each tetrahedra of the tessellation [16, 17]. Once the tessellation is built in the initial state of the simulation, it must remain intact through the whole evolution because of the Liouville's theorem, as long as the thermal velocities of the DM particles are vanishing. This requirement results in a significant difference between this approach and Delaunay tessellation suggested in [18] for estimating the density from particle distributions.

The particles being the vertices of the tetrahedra describe all deformations occurred to the geometry of the

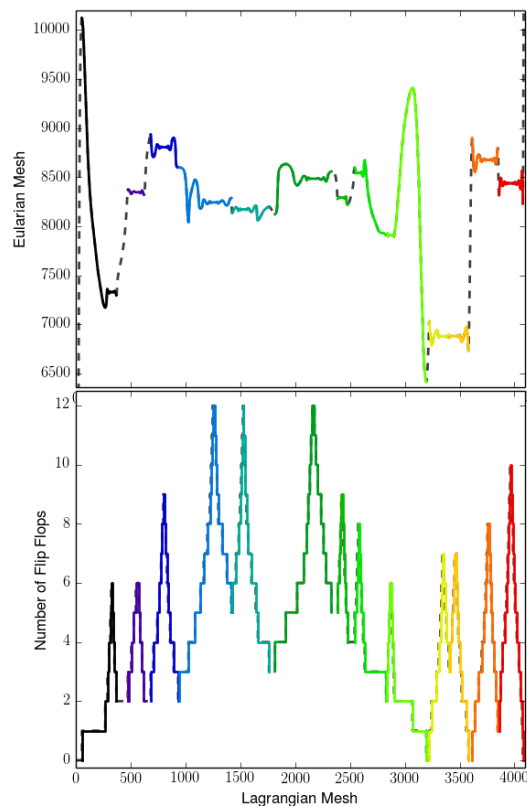


FIG. 2: Fields  $x(q)$  and  $n_H(q)$  are plotted in the top and bottom panels respectively for the halo shown in fig. 1.

tessellation. However it remains continuous in both six-dimensional phase space  $(\mathbf{x}, \mathbf{v})$  and in  $(\mathbf{q}, \mathbf{x})$  space. In particular, the variations of tetrahedra volumes result in the corresponding change of the tetrahedra densities. This property is especially valuable because it makes the tessellation self-adaptive to the growth of density perturbations with time. We stress that whereas both  $(\mathbf{x}, \mathbf{v})$  and  $(\mathbf{q}, \mathbf{x})$  spaces contain all the information about a dynamical system, the latter is a *metric* space and hence superior to the non-metric phase space. Moreover, the Lagrangian submanifold mapping,  $\mathbf{q} = \mathbf{q}(\mathbf{x})$ , is a single-valued function, unlike the phase-space mappings  $\mathbf{v} = \mathbf{v}(\mathbf{x})$  or  $\mathbf{x} = \mathbf{x}(\mathbf{v})$  which are multivalued.

We now illustrate the main idea of the proposed Lagrangian submanifold technique with a halo formed in one-dimensional  $N$ -body simulation of a collisionless cold DM medium in an expanding universe.

Figure 1 shows the phase space of the halo evolved in the universe from smooth random initial condition. The halo can be naturally defined as the region in Eulerian space where the number of streams is greater than one. The number of stream changes by two at caustics where the tangent to the phase space curve becomes vertical and the density in the corresponding stream becomes formally infinite. One can see a complicated substructure that consists of a number of subhaloes and streams

shown by different colors. It is obvious from the figure that identifying individual subhaloes in the configuration (Eulerian) space is difficult even in a simple one-dimensional model due to projection effects and the presence of tidal streams. It becomes even more challenging in three-dimensional simulations, see e.g., [13, 14] and reference therein.

Let us follow along the phase space curve in fig. 1 starting from the top point of the spiral on the left boundary of the box through the bottom point on the right boundary of the box. Along this path, the initial (Lagrangian) coordinates  $q_i$  of the particles, which are in essence their IDs, increase monotonically while their final (Eulerian) coordinates  $x_i$  are not monotonic as is also seen in the top panel of fig. 2. In other words there are fluid elements with  $x_{i+1} < x_i$  while  $q_{i+1} > q_i$ . We will dub every swap of the coordinates of the two neighboring particles on the curve as a flip-flop. The analog of this phenomenon in a multi-dimensional space is a formal change of the sign of the volume of a fluid particle when it turns inside out. The total number of flip-flops experienced by every fluid particle is shown in the bottom panel of fig. 2. Colors show individual peaks of the flip-flop field in Lagrangian coordinates. The correspondence of the flip-flop peaks in Lagrangian space to the individual subhaloes in the phase space is remarkable. Note that the tidal streams and their progenitor halos are also easily, unambiguously and robustly identified via the flip-flop field, cf. the bottom panel of figure 2 and the phase-space figure 1.

Next we show that the flip-flop field  $n_{\text{ff}}(\mathbf{q})$  exhibits similar features in a generic three-dimensional  $N$ -body simulation. The Lagrangian submanifold technique was implemented in the publicly available cosmological TreePM/SPH code GADGET [19] to compute the flip-flop field. The flop-flop module works as follows. At each time step and for each particle, the Jacobian  $J(\mathbf{q}, t) = |\partial x_i / \partial q_j|$  is evaluated and compared it with its value at the previous time step. If the sign of the Jacobian changes, the number of flip-flops for this particle is increased by one.

Initial conditions were generated with N-GenIC code with the standard  $\Lambda$ CDM cosmology,  $\Omega_m = 0.3, \Omega_\Lambda = 0.7, \Omega_b = 0, \sigma_8 = 0.9, h = 0.7$  and the initial redshift  $z = 50$ . A set of simulations were carried out with boxes ranging from  $100h^{-1}$  to  $1h^{-1}$  Mpc. For illustration purposes, we present here a relatively small zoomed-in simulation with  $256^3$  DM particles in a box with the comoving size of  $1h^{-1}$  Mpc with the force resolution of  $0.75h^{-1}$  kpc. The chosen size of the box is obviously too small for the purpose of deriving statistically valid properties of the haloes. However the main purpose of this work is different, namely to demonstrate that the flip-flop field of haloes in a highly nonlinear dynamic state retains rich information about the substructure in haloes.

Figure 3 shows the flip-flop field in Eulerian space with  $n_{\text{ff}}$  from 6 to 420, the maximum number of flip-flops in

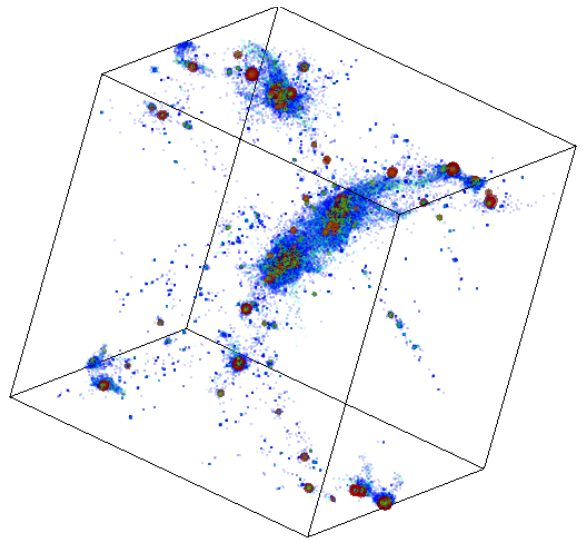


FIG. 3: The dot plot of the structure in the simulation of  $1/h$  Mpc box in the  $\Lambda$ CDM cosmology at  $z = 0$ . The sizes and colors from blue to red corresponds to the range  $n_{\text{ff}} \geq 6$ .

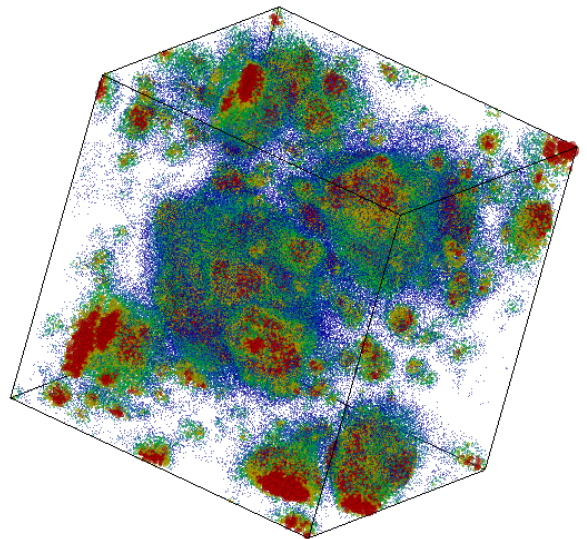


FIG. 4: The dot plot of the flip-flop field in Lagrangian space. The particles sizes and colors are similar to that in fig. 4.

this simulation. Obviously, this flip-flop field traces the distribution of matter in the universe. The sizes and colors (from blue to red) of the particles represent the number of flip-flops [20]. A dedicated analysis shows that the clumps of red particles (i.e., those with large flip-flop numbers) are not individual subhalos, but instead belong to different subhalos in the Lagrangian space, fig. 4. This indicates the inability of a configuration space-based analysis to disentangle all the substructure.

Figure 4 shows the corresponding flip-flop field in Lagrangian space with the same color coding. One can

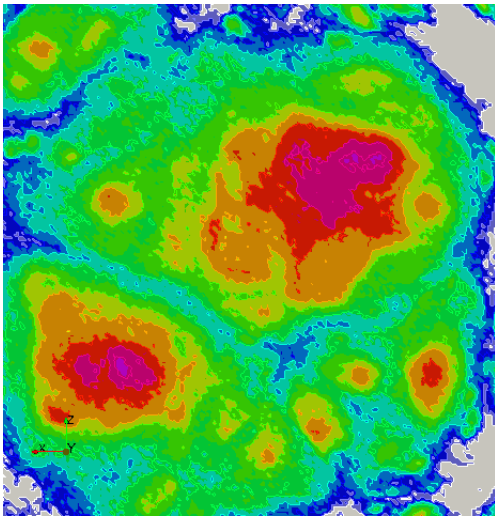


FIG. 5: The contour plot of  $n_{\text{ff}}(q)$  field on a 2D cross-section plane through the center of the cube shown in fig. 4. Contours from gray to magenta are:  $n_{\text{ff}} = 1.5, 2.5, 4, 7, 12, 27, 45, 75, 120$ , and 175.

clearly see that they form a large number of distinct flip-flop peaks in Lagrangian space. In order to reveal the much greater richness and complexity of the structure of subhaloes in the flip-flop field, we also plot a two-dimensional slice through Lagrangian space in fig. 5. This figure shows a complex hierarchy of peaks in greater detail. We also stress that the regions with zero flip-flops are, by definition, voids. Thus, the flip-flop formalism is a superior void finder, as it does not suffer from the poor density contrast or other issues.

A similar analysis as a function of redshift suggests that there are at least two distinct stages in the evolution of the flip-flop field: ‘fast and early’ and ‘slow and late’. Indeed, starting from the onset of nonlinearity, the flip-flop field evolves rapidly at  $z \gtrsim 1$  and slows down afterwards. The similarity and difference of the flip-flop field evaluated at different redshifts can be quantified by computing the correlation coefficient between the fields at two different epochs. Figure 6 shows the correlation coefficient for several pairs of the flip-flop fields. The lower line shows the correlation coefficient between the field at  $a = 1$  ( $z = 0$ ) and the fields at several previous stages. The top point on every curve shows the correlation coefficient of the field with itself and thus its value is one. The correlation coefficient monotonically decreases with the separation between the epochs. At first five stages shown, it steadily decreases which means that new peaks in the flip-flop field keep forming after the previous epoch. In contrast, the bottom four curves ‘pile up’, which suggests that the evolution slows down significantly.

Summarizing the results we conclude that the flip-flop field carries wealth of information about the substructures

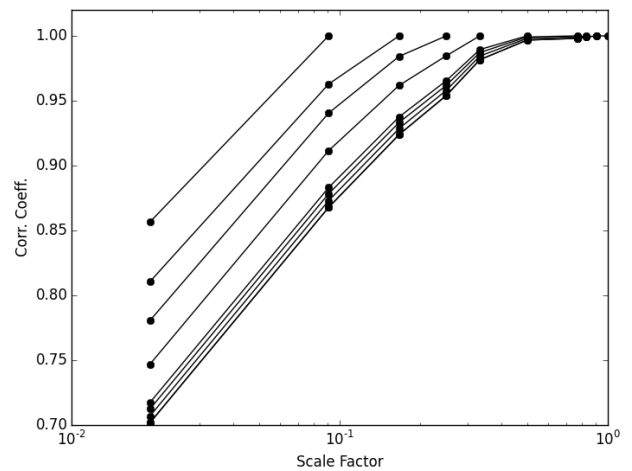


FIG. 6: The correlation coefficient between the flip-flop fields at different epochs. Every flip-flop field is correlated with several previous nonlinear stages. The top point on each curve marks the correlation coefficient of the field with itself and thus is exactly unity.

in the Cosmic Web in the form of peaks separated by the valleys with lower counts of flip-flops, as is illustrated in fig. 5. The peaks often consists of several higher peaks which in turn may consist of even a higher peak, forming a nesting structure resembling a Russian doll or ‘matryoshka’-doll. The topography of the flip-flop landscape evolves rapidly after the onset of nonlinearity marked by the origin of the first regions with  $n_{\text{ff}} > 0$ . Then its evolution seems to freeze or considerably slow down (see fig. 6) despite the peak heights continue to grow, which indicates ongoing rapid dynamics inside the halos themselves. Qualitatively similar pattern is also observed in two-dimensional case (not discussed here). We believe that the suggested method represents a valuable addition to the suite of various techniques suggested for studies of substructures in the Cosmic Web, see [12–14] and references therein.

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